DISCRETE-TIME SIGNALS AND SYSTEMS, PART 2

Solution 3.1

To correspond to a stable system, the unit sample response must be absolutely summable. For each system

$$\sum_{n=-\infty}^{+\infty} |h(n)| \text{ is given by:}$$

(i) 1

(ii) $\sum_{n=0}^{\infty} (\frac{1}{2})^n = 2$

(iii) $\sum_{n=-\infty}^{0} 2^n = \sum_{n=0}^{\infty} (\frac{1}{2})^n = 2$

Solution 3.2

(a) Consider the convolution sum in the form

$$y(n) = \sum_{k=-\infty}^{+\infty} x(k)\ h(n-k)$$

If $h(n) = 0,\ n < 0$, then $h(n-k) = 0,\ k > n$ and consequently

$$y(n) = \sum_{k=-\infty}^{n} x(k)\ h(n-k) \quad .$$

Thus $y(n_0)$ is given by

$$y(n_0) = \sum_{k=-\infty}^{n_0} x(k)\ h(n_0-k)$$

and hence depends only on values of $x(k)$ for $k \leq n_0$.

(b) If $h(n)$ is not zero for $n < 0$, then an example of an input for which the output anticipates the input is, of course, a unit sample. Alternatively, consider the convolution sum

$$y(n) = \sum_{k=-\infty}^{+\infty} x(k)\ h(n-k)$$

or,

$$y(n) = \sum_{k=-\infty}^{n} x(k)\ h(n-k) + \sum_{k=n+1}^{\infty} x(k)\ h(n-k)$$

If h(n) is not zero for $n < 0$ then at least one of the terms in the summation $\sum_{k=n+1}^{+\infty} x(k)\, h(n-k)$ will be non-zero, i.e. y(n) will depend on at least one value in x(k) for $k > n$.

Solution 3.3

Not all of these systems correspond to LSI systems. For those that don't we can examine stability and causality by referring back to the basic definitions.

(a) Clearly causal. Also stable since

$$|y(n)| = |g(n)|\,|x(n)|$$

Thus, if x(n) is bounded then y(n) will be bounded

(b) Not causal, since for $n < n_0$, y(n) will depend on values in x(k) for $k > n$. Also, not stable, since if $x(n) = 1$ for all n then $y(n) = (n - n_0 + 1)$ which is unbounded.

(c) Clearly stable. It is causal if $n_0 \geq 0$ and not causal otherwise.

Solution 3.4

(a) Rewriting the difference equation, with $x(n) = \delta(n)$ and h(n) denoting the unit-sample response

$$h(n) = \frac{1}{2}\, h(n-1) + \delta(n) + \frac{1}{2}\,\delta(n-1) \quad .$$

Since the system is causal, h(n) is zero for $n < 0$. For $n \geq 0$,

$$h(0) = \frac{1}{2}\, h(-1) + \delta(0) + \frac{1}{2}\,\delta(-1) = 1$$

$$h(1) = \frac{1}{2}\, h(0) + \delta(1) + \frac{1}{2}\,\delta(0) = 1$$

$$h(2) = \frac{1}{2}\, h(1) + \delta(2) + \frac{1}{2}\,\delta(1) = \frac{1}{2}$$

$$h(n) = 2\left(\frac{1}{2}\right)^n \qquad n \geq 1$$

h(n) can also be expressed as

$$h(n) = \left(\frac{1}{2}\right)^n \left[u(n) + u(n-1)\right] \quad .$$

(b) Substituting x(n) and h(n) into the convolution sum we obtain

$$y(n) = \sum_{k=-\infty}^{\infty} (\tfrac{1}{2})^k \left[u(k) + u(k-1)\right] e^{j\omega(n-k)}$$

$$= e^{j\omega n} \left[\sum_{k=0}^{\infty} (\tfrac{1}{2})^k e^{-j\omega k} + \sum_{k=1}^{\infty} (\tfrac{1}{2})^k e^{-j\omega k}\right]$$

$$= e^{j\omega n} \left[\frac{1 + \frac{1}{2} e^{-j\omega}}{1 - \frac{1}{2} e^{-j\omega}}\right]$$

(c) The frequency response of the system is the complex amplitude of the response with an excitation $e^{j\omega n}$. Thus, from part (a)

$$H(e^{j\omega}) = \frac{1 + \frac{1}{2} e^{-j\omega}}{1 - \frac{1}{2} e^{-j\omega}}$$

(d) With $H(e^{j\omega})$ expressed in polar form as $|H(e^{j\omega})|\, e^{j\Theta(\omega)}$, the response to the specified input is

$$y(n) = |H(e^{j\frac{\pi}{2}})| \cos(\tfrac{\pi n}{2} + \tfrac{\pi}{4} + \Theta(\tfrac{\pi}{2}))$$

From part (c),

$$|H(e^{j\frac{\pi}{2}})| = 1$$

$$\Theta(\tfrac{\pi}{2}) = -2 \tan^{-1}(\tfrac{1}{2})$$

Solution 3.5*

(a) If $x(n) = \delta(n)$ then the system response is $y(n) = a^n u(n)$. If $x(n) = \delta(n-1)$ then for $n > 0$ the system response will be $(a^n + a^{n-1})$. Therefore the system is not shift-invariant.

(b) The system is not linear, as can be demonstrated in a variety of ways. For example, a linear system has the property that $T[a\, x(n)] = aT[x(n)]$. Therefore if the input is doubled (for example) the output must double at each value of n, which for this system cannot happen at $n = 0$.

Solution 3.6*

The system excitation is $x(n) = e^{j\omega n}$. With ω replaced by $(\omega + 2\pi)$ the excitation becomes $e^{j(\omega+2\pi)n} = e^{j\omega n} e^{j2\pi n} = e^{j\omega n}$ and hence the input is periodic in ω. This means, in essence that in considering complex exponential inputs, a variation in ω over a range of 2π generates all of the distinguishable discrete-time complex exponentials. When ω varies outside that range we simply see the same ones over again. For the system described in this problem, the output for a given input is unique. Thus since $e^{j\omega n}$ and $e^{j(\omega+2\pi)n}$ are identical input signals their outputs must be identical, and consequently the output parameter P will be periodic with period 2π.

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