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 THE DISCRETE FOURIER SERIES
 

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 Solution 8.1
 

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$$\begin{aligned}\tilde{X}_1(k) &= \sum_{n=0}^{N-1} \tilde{x}(n) e^{-j\frac{2\pi}{N}nk} \\ &= 2 + e^{-j\frac{\pi}{2}k} + e^{-j\frac{\pi}{2}3k}\end{aligned}$$

$$e^{-j\frac{3\pi}{2}k} = e^{-jk(-\frac{3\pi}{2} + 2\pi)} = e^{j\frac{\pi}{2}k}$$

Therefore,

$$\tilde{X}_1(k) = 2 + e^{-j\frac{\pi}{2}k} + e^{j\frac{\pi}{2}k} = 2 \left[ 1 + \cos \frac{\pi k}{2} \right].$$

 Solution 8.2
 

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$$\begin{aligned}\tilde{X}(k) &= \sum_{n=0}^{N-1} \tilde{x}(n) W_N^{kn} \\ \tilde{X}^*(k) &= \sum_{n=0}^{N-1} \tilde{x}^*(n) W_N^{-kn}\end{aligned}$$

or, since  $\tilde{x}(n)$  is real,

$$\tilde{X}^*(k) = \sum_{n=0}^{N-1} \tilde{x}(n) W_N^{-kn}$$

Finally, substituting  $-k$  for  $k$

$$\tilde{X}^*(-k) = \sum_{n=0}^{N-1} \tilde{x}(n) W_N^{kn} = \tilde{X}(k)$$

Note, incidentally, that this is indeed satisfied for problem 8.1.

 Solution 8.3
 

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If we show that  $\tilde{X}(k)$  is real, then from problem 8.2 it follows that  $\tilde{X}(k)$  is also even. Thus

$$\tilde{X}^*(k) = \sum_{n=0}^{N-1} \tilde{x}(n) W_N^{-kn}$$

Replacing  $n$  by  $-n$  in the summation on the right-hand side

$$\tilde{X}^*(k) = \sum_{n=0}^{-N+1} \tilde{x}(-n) W_N^{kn}$$

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or since  $\tilde{x}(n)$  is even

$$\tilde{X}^*(k) = \sum_{n=0}^{-N+1} \tilde{x}(n) W_N^{kn}$$

Finally, since  $\tilde{x}(n)$  is periodic the limits on the summation can be replaced by the interval 0 to N-1. Thus  $\tilde{X}^*(k) = \tilde{X}(k)$ , i.e.  $\tilde{X}(k)$  is real.

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#### Solution 8.4

(i) Since  $\tilde{x}(n)$  is periodic with period 10,  $\tilde{X}(k)$  is also periodic with period 10. Thus (i) is true.

(ii) Since  $\tilde{x}(n)$  is real,  $\tilde{X}^*(k) = \tilde{X}(-k)$ . In order for the stated property to also be true,  $\tilde{X}(k)$  must be real, which requires that  $\tilde{x}(n)$  be even, which is not the case. Thus (ii) is not true.

(iii)  $\tilde{X}(0) = \sum_{n=0}^{N-1} \tilde{x}(n) = 0$ . Thus (iii) is true.

(iv)  $\tilde{X}(k) e^{jk \frac{2\pi}{5}}$  is the Fourier series for  $\tilde{x}(n+2)$ . From the figure we note that  $\tilde{x}(n+2)$  is not an even function. Thus  $\tilde{X}(k) e^{jk \frac{2\pi}{5}}$  is not real. However,  $\tilde{x}(n-2)$  is an even sequence and thus  $\tilde{X}(k) e^{-jk \frac{2\pi}{5}}$  is real

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#### Solution 8.5

See Figure S8.5-1 on next page.

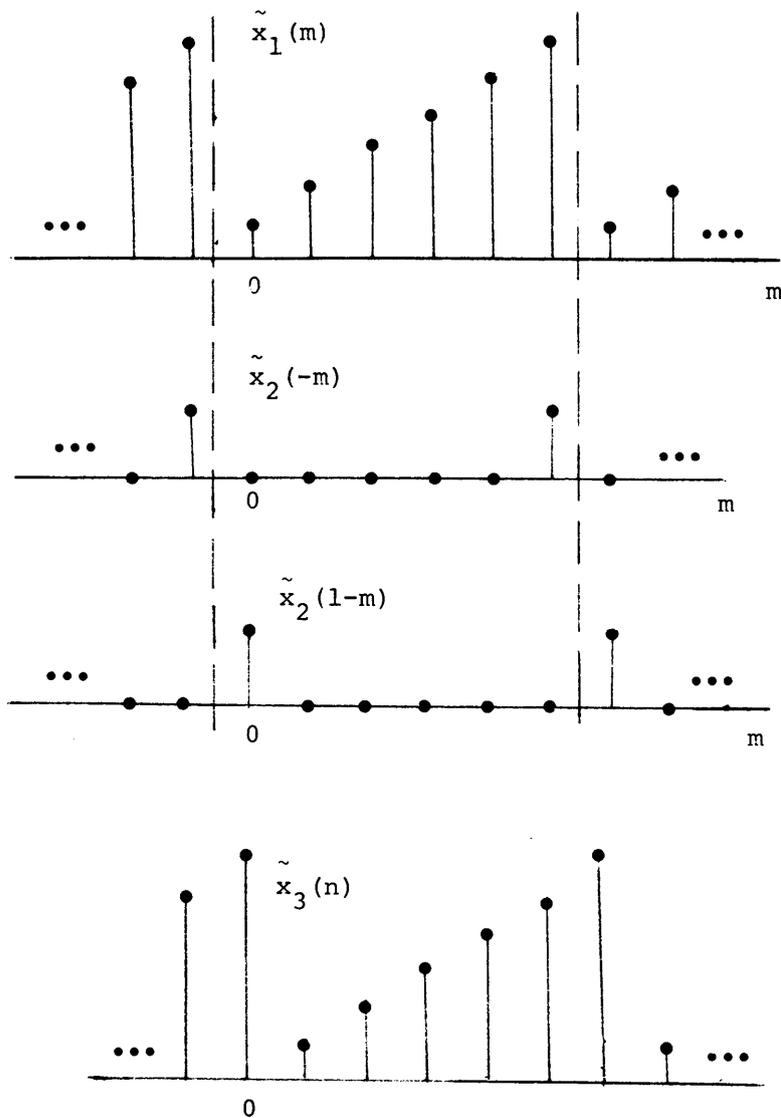


Figure S8.5-1

Solution 8.6

The Discrete Fourier series coefficients of  $\tilde{X}(k)$  would be defined as

$$\tilde{Y}(n) = \sum_{k=0}^{N-1} \tilde{X}(k) w_N^{kn}$$

$\tilde{x}(n)$  is given by

$$\tilde{x}(n) = \frac{1}{N} \sum_{k=0}^{N-1} \tilde{X}(k) w_N^{-kn}$$

thus  $\tilde{y}(n) = N \tilde{x}(-n)$ .

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Solution 8.7

(a) The time origin can be chosen such that all the  $\tilde{X}(k)$  are real if  $x(n)$  can be shifted to be an even function. It can for sequence (b) but not for the others.

(b) This requires that the time origin be chosen so that  $\tilde{x}(n)$  is odd. This cannot be done for any of the sequences.

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Solution 8.8

$$\begin{aligned}\tilde{X}_1(k) &= \sum_{n=0}^{N-1} \tilde{x}(n) W_N^k \\ \tilde{X}_2(k) &= \sum_{n=0}^{2N-1} \tilde{x}(n) W_{2N}^{kn} \\ &= \sum_{n=0}^{N-1} \tilde{x}(n) W_{2N}^{kn} + \sum_{n=0}^{N-1} \tilde{x}(n+N) W_{2N}^{k(n+N)}\end{aligned}$$

or, since  $\tilde{x}(n)$  is periodic with period  $N$  and  $W_{2N}^N = -1$

$$\begin{aligned}\tilde{X}_2(k) &= \sum_{n=0}^{N-1} \tilde{x}(n) W_{2N}^{kn} [1 + (-1)^k] \\ &= [1 + (-1)^k] \sum_{n=0}^{N-1} \tilde{x}(n) W_{2N}^{kn}\end{aligned}$$

Thus, for  $k$  odd,  $\tilde{X}_2(k) = 0$ . For  $k$  even,  $W_{2N}^{kn} = W_N^{n(k/2)}$

and

$$\begin{aligned}\tilde{X}_2(k) &= 2 \sum_{n=0}^{N-1} \tilde{x}(n) W_N^{n(k/2)} \\ &= 2 \tilde{X}_1(k/2) \quad k \text{ even.}\end{aligned}$$

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