Solution 8.1

$$\tilde{X}_1(k) = \sum_{n=0}^{N-1} \tilde{x}(n)\, e^{-j\frac{2\pi}{N} nk}$$

$$= 2 + e^{-j\frac{\pi}{2} k} + e^{-j\frac{\pi}{2} 3k}$$

$$e^{-j\frac{3\pi}{2} k} = e^{-jk(-\frac{3\pi}{2} + 2\pi)} = e^{j\frac{\pi}{2} k}$$

Therefore,

$$\tilde{X}_1(k) = 2 + e^{-j\frac{\pi}{2} k} + e^{j\frac{\pi}{2} k} = 2 \left[1 + \cos \frac{\pi k}{2}\right].$$

Solution 8.2

$$\tilde{X}(k) = \sum_{n=0}^{N-1} \tilde{x}(n)\, W_N^{kn}$$

$$\tilde{X}^*(k) = \sum_{n=0}^{N-1} \tilde{x}^*(n)\, W_N^{-kn}$$

or, since $\tilde{x}(n)$ is real,

$$\tilde{X}^*(k) = \sum_{n=0}^{N-1} \tilde{x}(n)\, W_N^{-kn}$$

Finally, substituting $-k$ for k

$$\tilde{X}^*(-k) = \sum_{n=0}^{N-1} \tilde{x}(n)\, W_N^{kn} = \tilde{X}(k)$$

Note, incidentally, that this is indeed satisfied for problem 8.1.

Solution 8.3

If we show that $\tilde{X}(k)$ is real, then from problem 8.2 it follows that $\tilde{X}(k)$ is also even. Thus

$$\tilde{X}^*(k) = \sum_{n=0}^{N-1} \tilde{x}(n)\, W_N^{-kn}$$

Replacing n by -n in the summation on the right-hand side

$$\tilde{X}^*(k) = \sum_{n=0}^{-N+1} \tilde{x}(-n)\, W_N^{kn}$$

or since $\tilde{x}(n)$ is even

$$\tilde{X}^*(k) = \sum_{n=0}^{-N+1} \tilde{x}(n)\, W_N^{kn}$$

Finally, since $\tilde{x}(n)$ is periodic the limits on the summation can be replaced by the interval 0 to N-1. Thus $\tilde{X}^*(k) = \tilde{X}(k)$, i.e. $\tilde{X}(k)$ is real.

Solution 8.4

(i) Since $\tilde{x}(n)$ is periodic with period 10, $\tilde{X}(k)$ is also periodic with period 10. Thus (i) is true.

(ii) Since $\tilde{x}(n)$ is real, $\tilde{X}^*(k) = \tilde{X}(-k)$. In order for the stated property to also be true, $\tilde{X}(k)$ must be real, which requires that $\tilde{x}(n)$ be even, which is not the case. Thus (ii) is not true.

(iii) $\tilde{X}(0) = \sum_{n=0}^{N-1} \tilde{x}(n) = 0$. Thus (iii) is true.

(iv) $\tilde{X}(k)\, e^{jk\frac{2\pi}{5}}$ is the Fourier series for $\tilde{x}(n + 2)$. From the figure we note that $\tilde{x}(n + 2)$ is not an even function. Thus $\tilde{X}(k)\, e^{j\frac{k2\pi}{5}}$ is not real. However, $\tilde{x}(n - 2)$ is an even sequence and thus $\tilde{X}(k)\, e^{\frac{-jk2\pi}{5}}$ is real

Solution 8.5

See Figure S8.5-1 on next page.

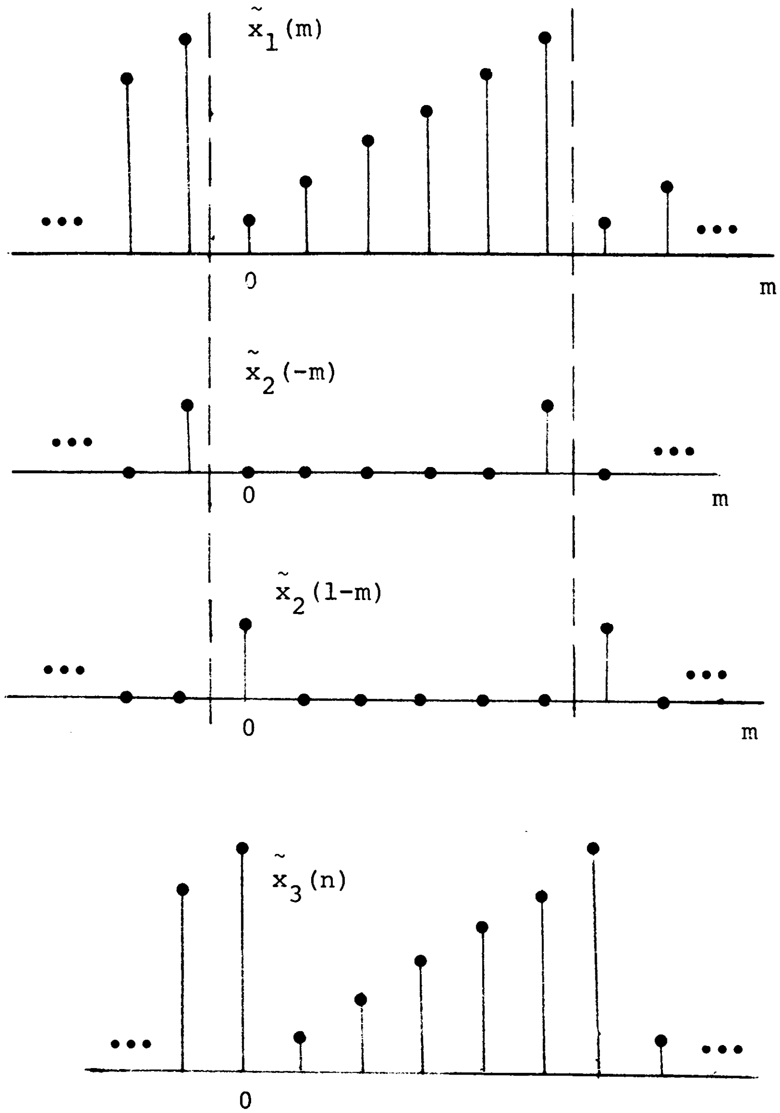

Figure S8.5-1

Solution 8.6

The Discrete Fourier series coefficients of $\tilde{X}(k)$ would be defined as

$$\tilde{y}(n) = \sum_{k=0}^{N-1} \tilde{X}(k) W_N^{kn}$$

$\tilde{x}(n)$ is given by

$$\tilde{x}(n) = \frac{1}{N} \sum_{k=0}^{N-1} \tilde{X}(k)\ W_N^{-kn}$$

thus $\tilde{y}(n) = N\ \tilde{x}(-n)$.

Solution 8.7

(a) The time origin can be chosen such that all the $\tilde{X}(k)$ are real if $x(n)$ can be shifted to be an even function. It can for sequence (b) but not for the others.

(b) This requires that the time origin be chosen so that $\tilde{x}(n)$ is odd. This cannot be done for any of the sequences.

Solution 8.8

$$\tilde{X}_1(k) = \sum_{n=0}^{N-1} \tilde{x}(n)\ W_N^{k}$$

$$\tilde{X}_2(k) = \sum_{n=0}^{2N-1} \tilde{x}(n)\ W_{2N}^{kn}$$

$$= \sum_{n=0}^{N-1} \tilde{x}(n)\ W_{2N}^{kn} + \sum_{n=0}^{N-1} \tilde{x}(n+N)\ W_{2N}^{k(n+N)}$$

or, since $\tilde{x}(n)$ is periodic with period N and $W_{2N}^{N} = -1$

$$\tilde{X}_2(k) = \sum_{n=0}^{N-1} \tilde{x}(n)\ W_{2N}^{kn}\ [1 + (-1)^k]$$

$$= [1+(-1)^k] \sum_{n=0}^{N-1} \tilde{x}(n)\ W_{2N}^{kn}$$

Thus, for k odd, $\tilde{X}_2(k) = 0$. For k even, $W_{2N}^{kn} = W_N^{n(k/2)}$

and

$$\tilde{X}_2(k) = 2 \sum_{n=0}^{N-1} \tilde{x}(n)\ W_N^{n(k/2)}$$

$$= 2\ \tilde{X}_1(k/2) \qquad\qquad k \text{ even.}$$