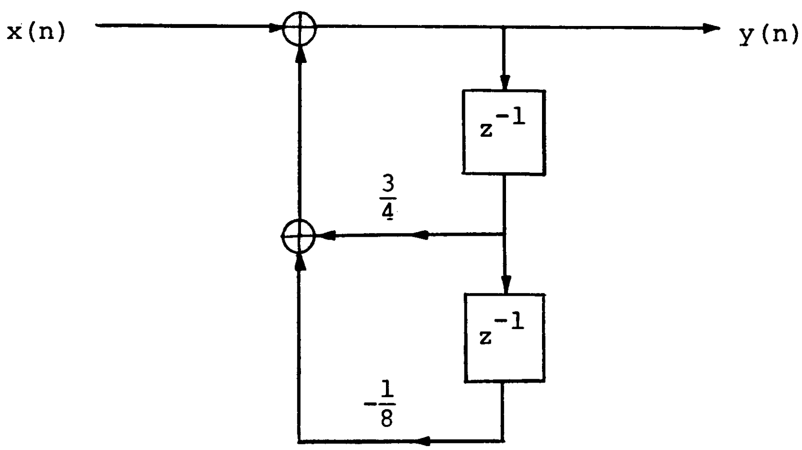
REPRESENTATION OF LINEAR DIGITAL NETWORKS

## Solution 11.1

(a)




(b)

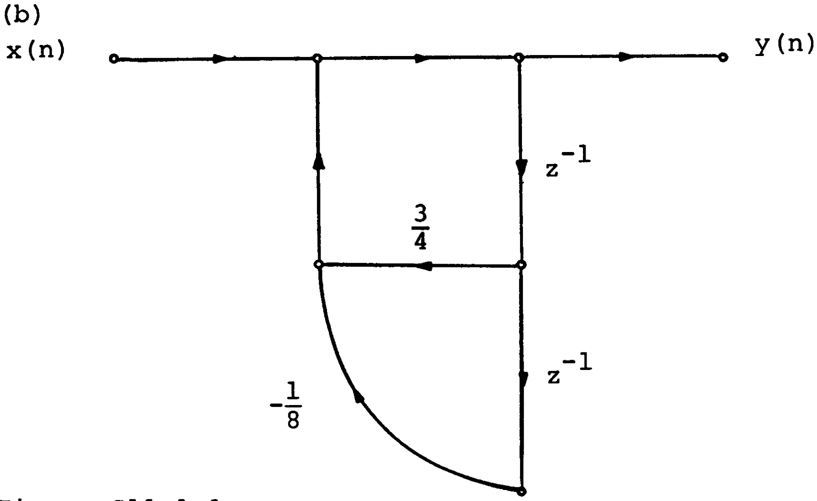



Figure S11.1-1

We have drawn the flow-graph to graphically correspond closely to the block diagram in (a). There are, of course, many other ways of drawing the flow-graph, for example

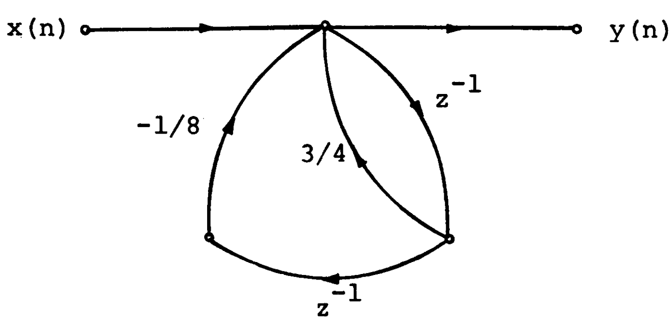



Figure S11.1-2

## Solution 11.2

(a) The equations corresponding to this flowgraph are:

$W_1(z) = X(z)$

$W_2(z) = W_1(z) + z^{-1} W_4(z)$

$W_3(z) = 2 W_2(z)$

$W_4(z) = 2 W_1(z) + 3 W_3(z)$

$Y(z) = W_3(z)$

or, in matrix form

$$\begin{bmatrix} W_1 \\ W_2 \\ W_3 \\ W_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & z^{-1} \\ 0 & 2 & 0 & 0 \\ 2 & 0 & 3 & 0 \end{bmatrix} \begin{bmatrix} W_1 \\ W_2 \\ W_3 \\ W_4 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} X(z)$$

$$Y(z) = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} W_1 \\ W_2 \\ W_3 \\ W_4 \end{bmatrix}$$

$$\underline{F}_c^{\,t} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 2 & 0 & 3 & 0 \end{bmatrix}$$

and

$$\underline{F}_d^{\,t} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

## Solution 11.3

Let us carry this out by obtaining the transfer function for each of the networks. For network 1:

$$Y(z) = 2r \cos\theta\, z^{-1} Y(z) - r^2 z^{-2} Y(z) + X(z)$$

or

$$H_1(z) = 1/[1 - 2r \cos\theta\, z^{-1} + r^2 z^{-2}]$$

For network 2:
Define $W_1(z)$ as shown in Figure S11.3-1.

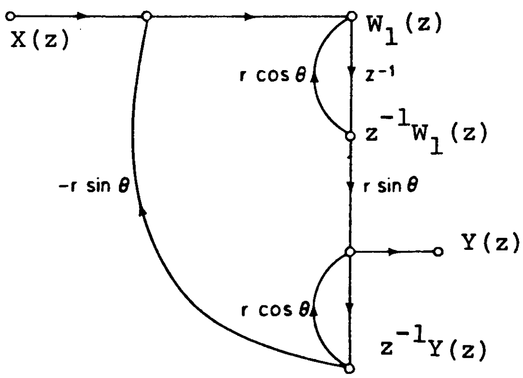



Figure S11.3-1

then

$$W_1(z) = X(z) - r \sin\theta\, z^{-1} Y(z) + r \cos\theta\, z^{-1} W_1(z)$$

$$Y(z) = r \sin\theta\, z^{-1} W_1(z) + r \cos\theta\, z^{-1} Y(z)$$

Solving for $Y(z)$ in terms of $X(z)$ we obtain

$$Y(z) = X(z)\, r(\sin\theta)\, z^{-1}/[1 - 2r \cos\theta\, z^{-1} + r^2 z^{-2}]$$

or

$$H_2(z) = r(\sin\theta)\, z^{-1}/[1 - 2r \cos\theta\, z^{-1} + r^2 z^{-2}]$$

Thus both networks have the same poles.

## Solution 11.4

With the nodes ordered as shown in the figure, the matrix $\underline{F}_c^{\,t}$ is

$$\underline{F}_c^{\,t} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

Thus with the nodes arranged in the order 1-2-3-4 they can be computed in sequence since the matrix $\underline{F}_c^t$ is zero on and above the main diagonal. There is no other ordering possible.

Solution 11.5

(a) A flow-graph in terms of $H_1$, $H_2$, $H_3$ and $H_4$ can be drawn as

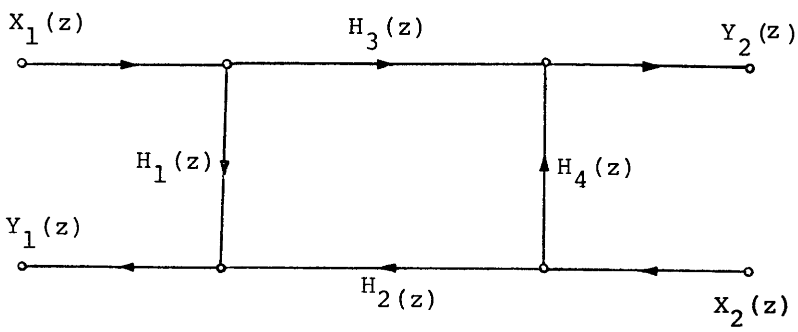



Figure S11.5-1

However we want to draw the flow-graph using branch transmittances which are constant or a constant times $z^{-1}$. Thus we replace $H_1$, $H_2$, $H_3$ and $H_4$ by their flow-graph implementations to obtain

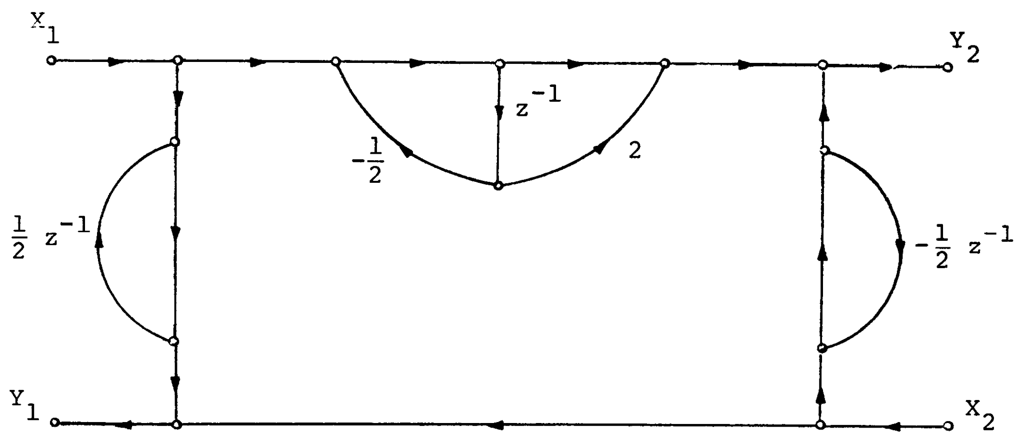



Figure S11.5-2

(b) We now want to connect a network $H_B(z)$ to the right-hand side of the above network. Observe that there is a delay-free path from $X_2$ to $Y_2$. Consequently, if the system $H_B(z)$ has a delay-free path from its input to its output, the total system will have a delay-free loop and thus will be noncomputable. By contrast, if $H_B(z)$ does not have a delay-free path from its input to its output, the overall system will be computable. A necessary and sufficient condition such that $H_B(z)$ is not delay-free from input to output is that $h_B(n)$, its unit sample response be zero at n = 0 (we are, of course, assuming that the system is causal.) This is guaranteed if $H_B(z)$ can be written in the form

$$H_B(z) = z^{-1}\,\hat{H}_B(z)$$

where $\hat{H}_B(z)$ is also causal, or equivalently that

$$\lim_{z\to\infty} H_B(z) = 0.$$