

COMPUTATION OF THE DISCRETE FOURIER TRANSFORM - PART 1

Solutions 18.1

The flow-graph of Fig. 9.3 of the text is based on the decomposition of $X(k)$ in the form of equation 9.14 of text. For $N=16$, the corresponding flow-graph expressing $X(k)$ as a combination of two eight-point DFT's is shown in Figure S18.1-1 below.

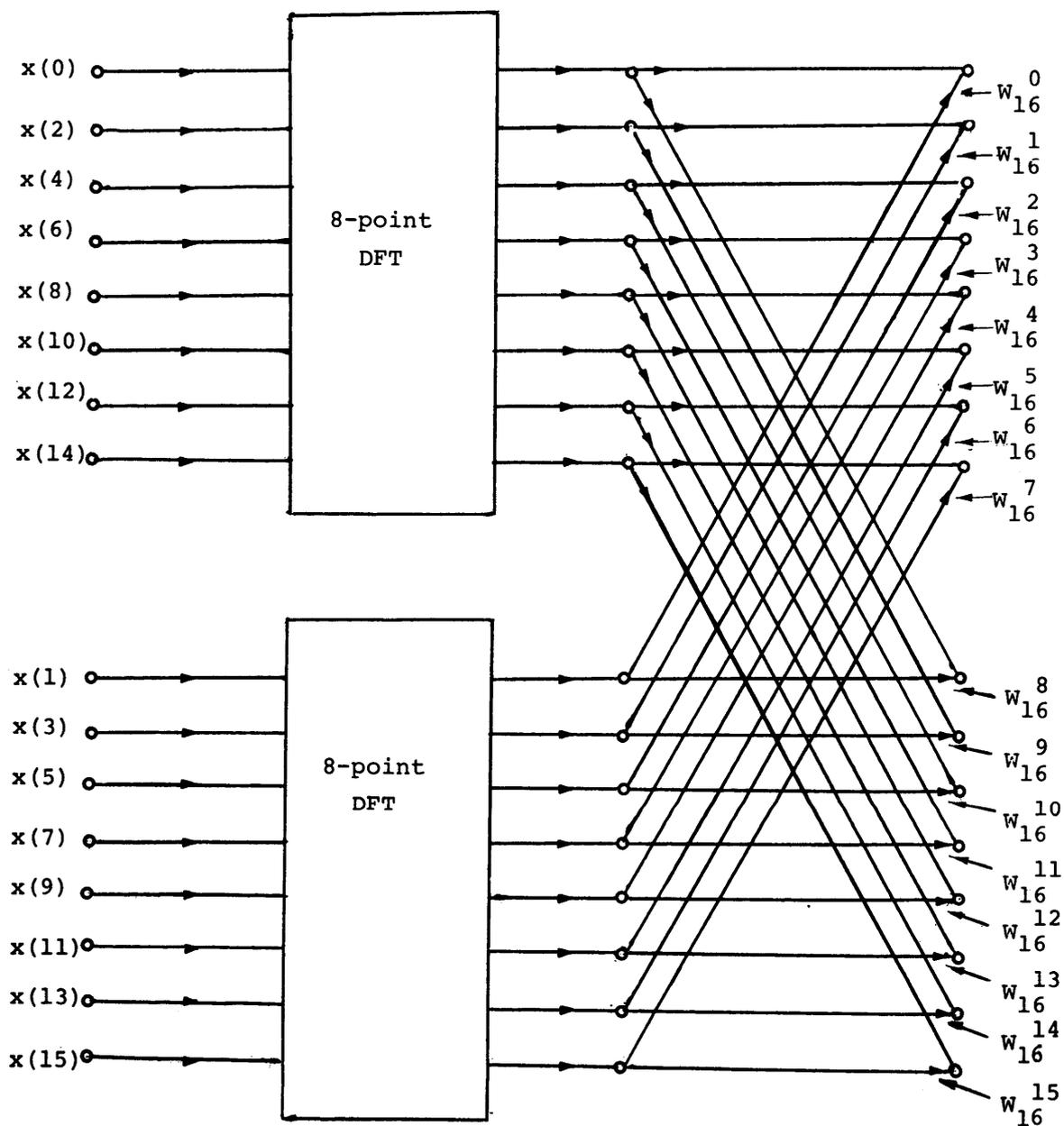


Figure S18.1-1

Solution 18.2

From the expression for the inverse DFT it follows that

$$N x^*(n) = \sum_{k=0}^{N-1} X^*(k) e^{-j\left(\frac{2\pi}{N}\right)kn}$$

Thus by using as the input to the DFT program the complex conjugate of $X(k)$, the output sequence will be N times the complex conjugate of $x(n)$.

Solution 18.3

(a) $\frac{N}{2}$

(b) In proceeding from array $(m-1)$ to array m we are combining $2^{(m-1)}$ point DFT's to form 2^m point DFT's. Thus the coefficients are successive powers of W_M where $M = 2^m$. Thus these coefficients are W_M^k where $k = 0, 1, 2, \dots, \left(\frac{M}{2} - 1\right)$ or, since

$$W_M = (W_N)^{N/M}$$

The powers of W_N involved in computing the m^{th} array from the $(m-1)$ st array are

$$W_N^{Nk/M} \quad k = 0, 1, 2, \dots, (M/2 - 1)$$

(c) $2^{(m-1)}$

(d) 2^m for $1 \leq m \leq (\log_2 N) - 1$. For the last array ($m = \log_2 N$) there are no two butterflies utilizing the same coefficients.

Solution 18.4

With $g(n) = x_1(n) + j x_2(n)$,

$$G(k) = X_1(k) + j X_2(k)$$

With $X_1(k)$ and $X_2(k)$ expressed in terms of their real and imaginary parts,

$$X_1(k) = X_{1R}(k) + j X_{1I}(k)$$

$$X_2(k) = X_{2R}(k) + j X_{2I}(k)$$

$G(k)$ can be written as

$$G(k) = [X_{1R}(k) - X_{2I}(k)] + j [X_{2R}(k) + X_{1I}(k)]$$

Now, since $x_1(n)$ and $x_2(n)$ are real, $X_{1R}(k)$ and $X_{2R}(k)$ are even and $X_{1I}(k)$ and $X_{2I}(k)$ are odd. Thus,

$$G_{ER}(k) = X_{1R}(k)$$

$$G_{OR}(k) = -X_{2I}(k)$$

$$G_{EI}(k) = X_{2R}(k)$$

$$G_{OI}(k) = X_{1I}(k)$$

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