COMPUTATION OF THE DISCRETE FOURIER TRANSFORM - PART 2

Solution 19.1

NORMAL ORDER	BIT REVERSED ORDER
x(0000) = x(0)	x(0000) = x(0)
x(0001) = x(1)	x(1000) = x(8)
x(0010) = x(2)	x(0100) = x(4)
x(0011) = x(3)	x(1100) = x(12)
x(0100) = x(4)	x(0010) = x(2)
x(0101) = x(5)	x(1010) = x(10)
x(0110) = x(6)	x(0110) = x(6)
x(0111) = x(7)	x(1110) = x(14)
x(1000) = x(8)	x(0001) = x(1)
x(1001) = x(9)	x(1001) = x(9)
x(1010) = x(10)	x(0101) = x(5)
x(1011) = x(11)	x(1101) = x(13)
x(1100) = x(12)	x(0011) = x(3)
x(1101) = x(13)	x(1011) = x(11)
x(1110) = x(14)	x(0111) = x(7)
x(1111) = x(15)	x(1111) = x(15)

Solution 19.2

(a)

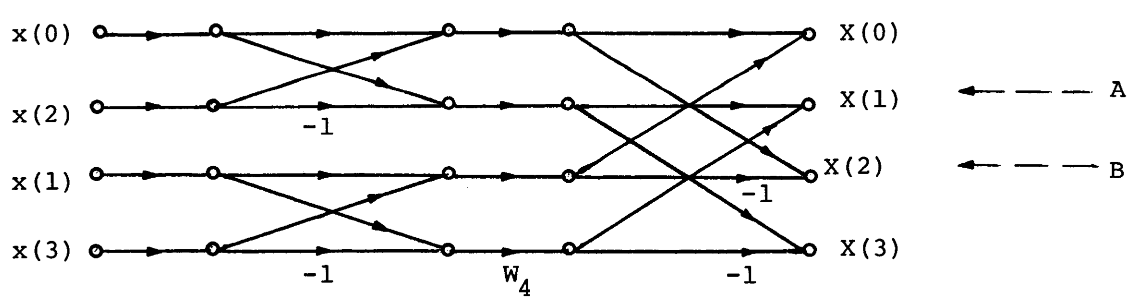

Figure S19.2-1

(b)

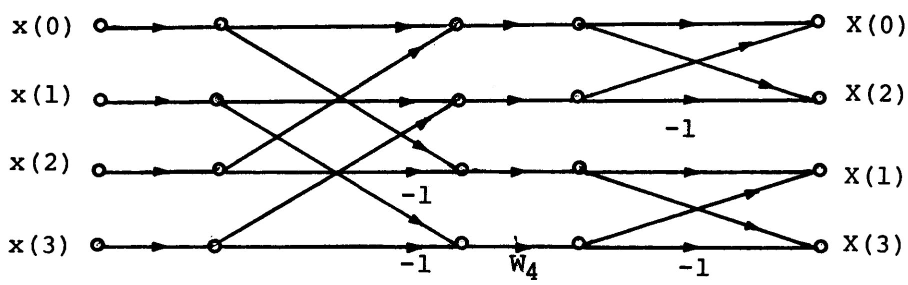

Figure S19.2-2

The desired flow-graph is obtained by interchanging lines A and B in Figure S19.2-1

Solution 19.3

(a) $N/2$

(b) $W_N^{Mk/2}$ $\quad k = 0,1,\ldots,N/M - 1 \qquad M = 2^m$

(c) $N\cdot 2^{-m}$

(d) $N\cdot 2^{-m+1}$; $\quad 2 \leq m \leq \log_2 N$

Solution 19.4*

(a) $X_{m+1}(p) = X_m(p) + W_N^r X_m(q)$

$$|X_{m+1}(p)| \leq |X_m(p)| + |W_N^r X_m(q)| = |X_m(p)| + |X_m(q)|$$

Thus with $|X_m(p)|$ and $|X_m(q)|$ both less than 1/2,

$|X_{m+1}(p)| < 1$

and consequently $|X_{m+1}(p)|^2 < 1$

Finally, since $|X_{m+1}(p)|^2 = \left\{\text{Re } [X_{m+1}(p)]\right\}^2 + \left\{\text{Im } [X_{m+1}(p)]\right\}^2$

it follows that

$|\text{Re } [X_{m+1}(p)]| < 1$

$|\text{Im } [X_{m+1}(p)]| < 1$

In a similar manner it follows that

$|\mathrm{Re}\ [X_{m+1}(q)]| < 1$

$|\mathrm{Im}\ [X_{m+1}(q)]| < 1$

(b) $\mathrm{Re}\ [X_{m+1}(p)] = \mathrm{Re}\ [X_m(p)] + \mathrm{Re}\ [W_N^r\ X_m(q)]$

Let $X_m(q)$ be expressed in polar form as $Ae^{j\theta}$

Then $\mathrm{Re}\ [X_{m+1}(p)] = \mathrm{Re}\ [X_m(p)] + A\cos(\theta - \frac{2\pi r}{N})$

$\mathrm{Re}\ [X_m(p)]$ is constrained to be less than 1/2 and since the magnitudes of the real and imaginary parts of $X_m(q)$ are constrained to be less than 1/2, A must be less than $1/\sqrt{2}$. If $\theta = \frac{2\pi r}{N}$, then

$|\mathrm{Re}\ [X_{m+1}(p)]| = |\mathrm{Re}\ [X_m(p)] + A| < \frac{1}{2} + \frac{1}{\sqrt{2}}$

Since this is greater than unity, we see that the stated constraints will not guarantee that no over-flow occurs.

Solution 19.5*

(a)

(1) Bit reversal - lines 7 through 16

(2) Recursive computation of W_N's - line 29

(3) Basic Butterfly computation - lines 26 through 28

(b)

(1) Insert between lines 7 and 8: `If (I.GE.J) GO TO 5`

(2) Line 22 should read:

```
W = CMPLX(COS(PI/FLOAT(LEI)), - SIN(PI/FLOAT(LE1)))
```

(3) Line 25: `IP = I + LE1`