

1 Introduction

Mathematical physics operates with a combination of the three major branches of mathematics: geometry, algebra and infinitesimal analysis. The interplay of these elements has undergone a considerable change since the turn of the century.

In classical physics, analysis, in particular differential equations, plays a central role. This formalism is supplemented most harmoniously by Gibbsian vector algebra and calculus to account for the spatial, geometric properties of particles and fields.

Few theorists have labored as patiently as Gibbs to establish the simplest possible formalism to meet a particular need. His success can be assessed by the fact that — almost a century later — his calculus, in the original notation, is in universal use. However, once full advantage has been taken of all simplifications permitted in the classical theory, there did not remain the reserve capacity to deal with quantum mechanics and relativity. The gap in the classical algebraic framework was supplemented by Minkowski tensors and Hilbert vectors, matrix algebras, spinors, Lie groups and a host of other constructs.

Unfortunately, the advantages derived from the increased power and sophistication of the new algebraic equipment are marred by side effects. There is a proliferation of overlapping techniques with too little standardization.

Also, while the algebras maintain a vaguely geometrical character, the “spaces” referred to are mathematical abstractions with but scant appeal to ordinary spatial intuition.

These features are natural consequences of rapid growth which can be remedied by consolidation and streamlining; the problem is to adapt the Gibbsian principle of economy to more demanding conditions.

This course is a progress report on such a project. Our emphasis on formalism does not mean neglect of conceptual problems. In fact, the most rewarding aspect of our consolidation is the resulting conceptual clarity.

The central idea of the present approach is that group theory provides us with a flexible and comprehensive framework to describe and classify fundamental physical processes. It is hardly necessary to argue that this method is indeed important in modern physics, yet its potentialities are still far from exhausted. This may stem from the tendency to resort to group theory only for difficult problems when other methods fail, or become too cumbersome. As a result, the discussions tend to be complicated and abstract.

The distinctive feature of the present method is to start with elementary problems, but treat them from an advanced view point, and to reconcile intuitive interpretation with a smooth transition to deeper problems. By focusing on group theory from the outset, we can make full use of its unifying power.

As mentioned above, this is a report on work in progress. Although we shall confine ourselves to problems that fall within the scope of the “consolidated” theory, we shall be in a position to discuss some of the conceptual problems of quantum mechanics.