

Unit Analysis

Problem Solving Series

Instructor's Guide

CONTENTS

INTRO

PHYS 101

RESOURCES

Table of Contents

Introduction	2
When to Use this Video	2
Learning Objectives	2
Motivation	2
Student Experience	2
Key Information	2
Video Highlights	3
Video Summary	3
Phys 101 Materials	4
Pre-Video Materials	4
Post-Video Materials	4
Additional Resources	6
Going Further	6
References	6
Appendix	A1

DEVELOPED BY THE TEACHING AND LEARNING LABORATORY AT MIT
FOR THE SINGAPORE UNIVERSITY OF TECHNOLOGY AND DESIGN



Introduction

When to Use this Video

- In Phys 101, at home or in recitation, before or during Module 1: Units and Significant Figures, Recitation 1
- Prior knowledge: formula for work

Learning Objectives

After watching this video students will be able to:

- Utilize and apply the key properties of unit analysis: when two quantities are multiplied, their units also multiply, and all terms added, subtracted in an equation must have the same units.
- Explain how derivatives and integrals affect units.

Motivation

- Units connect numbers to physical quantities.
- The basic ideas of unit analysis provide the foundations for dimensional analysis, which is extremely important in checking solutions, predicting formulas, and designing experiments.
- Even though unit analysis seems basic, many students struggle with these basic concepts, and fail to be consistent and careful in their work with units. Experts think about units at every step along the way as they solve problems, so developing good habits early on is important.

Student Experience

It is highly recommended that the video is paused when prompted so that students are able to attempt the activities on their own and then check their solutions against the video.

During the video, students will:

- Look for a mistake in the solution started with an incorrect approach.
- Identify the units of constants in given equations.
- Check the units of the problem solution to assess whether the solution is correct and the answer makes physical sense.

Key Information

Duration: 14:09

Narrator: Prof. Ken Kamrin

Materials Needed:

- Paper and pencil
- Calculator

CONTENTS

INTRO

PHYS 101

RESOURCES

Video Highlights

This table outlines a collection of activities and important ideas from the video.

Time	Feature	Comments
0:01	Mars orbiter example	Segment shows how units played important role in a NASA error.
1:25	Chapter 1: unit analysis and calculus	Works through how differentiation and integration effect units.
1:44	Differential quantity of ice cream	Segment providing an analogy for why dx has the same units as x through the use of smaller and smaller quantities of ice cream.
2:16	Differentiation and units	
3:37	Integration and units	
4:54	Chapter 2: example from physics	Presents and solves a problem from physics to compute the work done by a machine on an object. Uses unit analysis to check work along the way.
5:05	Animation of problem scenario	Maple animation of the position function of the object in the problem.
5:31	Work	Illustration of the concept of work.
9:35	Activity: identify units of the constants in formulas for velocity and acceleration	Students can solve directly, or use understanding of how derivatives effect units to determine the units of constants in two given formulas for velocity and acceleration obtained through differentiating the position function.
12:36	Activity: check units of final computation	Important application of the unit analysis skill.

CONTENTS

INTRO

PHYS 101

RESOURCES

Video Summary

This video has 2 chapters. The first chapter determines the units of the differential quantity dx in terms of the units of x , and explores how integration and differentiation effect units. The second chapter works through a physics example computing the work of an applied force on an object. The effects of derivatives and integrals are used to check the units of the physical quantities involved at several steps along the way.

Phys 101 Materials

Pre-Video Materials

When appropriate, this guide is accompanied by additional materials to aid in the delivery of some of the following activities and discussions.



1. Using some of the formulas that students know for energy, have students identify the units of energy in terms of mass in kilograms, distance in meters, and time in seconds.

Break students into small groups. Start by having student brainstorm different energy relationships and formulas. Then use one relationship to determine the units of energy in terms of mass, distance, and time. After they determine the units of energy, have them compare their solution and approach with other groups. Here are some questions to consider: did anyone use $E=mc^2$ in order to determine the units of energy?, what about kinetic energy?, potential energy?



2. Have students identify units of at least two quantities relevant to a field in which they are interested. Determine if there are any relations between these quantities, and if so, discuss how units can help determine necessary conditions on the relationships allowed.

Post-Video Materials

In each multiple choice question, the function $f(x)$ has units of “cars”, and the variable x has units of “people”. These questions are available in a file called “Unit Analysis Clicker Questions.pdf”.



1. Clicker Question (Appendix A1)

What are the units of $\frac{df}{dx}$?



- (a) Cars
- (b) Car-people
- (c) Cars per (people)²
- (d) Cars per person
- (e) Undefined



2. Clicker Question (Appendix A2)

What are the units of $\int f dx$?



- (a) Cars² per person
- (b) Car-people
- (c) Cars per (people)²
- (d) Cars per person
- (e) Undefined



3. Clicker Question (Appendix A3)

What are the units of $\frac{d^2 f}{dx^2}$?



- (a) Cars² per person
- (b) Car-people
- (c) Cars per (people)²
- (d) Cars per person
- (e) Undefined



4. Clicker Question (Appendix A4)

What are the units of $f(x) + x = 0$?



- (a) Cars² per person
- (b) Car-people
- (c) Cars per (people)²
- (d) Cars per person
- (e) Undefined

This problem may cause some difficulty if students forget that quantities in an equation must all have the same units. If students have difficulty, this issue can be further illuminated by the following discussion problem.



5. Have students describe why x must be unit-less whenever e^x appears in an equation.

Hint: Show this by linearizing e^x , and showing that if x were to have units, this linearization would allow you to “solve for the units” of x .

Additional Resources

Going Further

Unit analysis provides the basic tool kit for understanding how units behave mathematically. Once these foundations are set, they provide the framework for doing dimensional analysis. Dimensional analysis starts with physical quantities that have units, and combines them using multiplication, division, and powers in order to end up with a collection of independent dimensionless quantities. These dimensionless quantities can then be put into equations to model system behavior. Dimensional analysis is used heavily in creating scaled experiments.

References

The following MIT video lecture discusses units and dimensional analysis.

- Lewin, Walter. *8.01 Physics I: Classical Mechanics, Fall 1999*. (Massachusetts Institute of Technology: MIT OpenCourseWare), <http://ocw.mit.edu> (Accessed 5 Jan., 2012). License: Creative Commons BY-NC-SA
- Lecture 1: Units and Dimensional Analysis

The following article motivates the discussion of Unit Analysis and common student difficulties.

- Saitta, E. K., Gittings, M. J., and Geiger, C. (2011). Learning Dimensional Analysis through Collaboratively working with Manipulatives. *J. Chem. Ed.*, 88(7), 910-915.

The following resources contain problems, examples, and methods for using dimensional analysis.

- Mahajan, S. (2010). *Streetfighting Mathematics: the art of educated guessing and opportunistic problem solving*, MIT Press, Cambridge MA.
- Misic, T., Najdanovic-Lukic, M., and Nesic, L. (2010). Dimensional analysis in physics and the Buckingham theorem, *Eur. J. Phys.*, 31(4), 893-906.

- The function $f(x)$ has units of “cars”
- The variable x has units of “people”

1. What are the units of $\frac{df}{dx}$?

- a) Cars
- b) Car-People
- c) Cars per (People)²
- d) Cars per People
- e) Undefined

- The function $f(x)$ has units of “cars”
 - The variable x has units of “people”
2. What are the units of $\int f(x)dx$?
- a) Cars² per Person
 - b) Car-People
 - c) Cars per (People)²
 - d) Cars per Person
 - e) Undefined

- The function $f(x)$ has units of “cars”
- The variable x has units of “people”

3. What are the units of $\frac{d^2 f}{dx^2}$?

- a) Cars² per Person
- b) Car-People
- c) Cars per (People)²
- d) Cars per Person
- e) Undefined

- The function $f(x)$ has units of “cars”
 - The variable x has units of “people”
4. What are the units of $f(x) + x = 0$?
- a) Cars² per Person
 - b) Car-People
 - c) Cars per (People)²
 - d) Cars per Person
 - e) Undefined

MIT OpenCourseWare
<http://ocw.mit.edu>

RES.TLL.004 STEM Concept Videos
Fall 2013

For information about citing these materials or our Terms of Use, visit: <http://ocw.mit.edu/terms>.